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Burnside problems for Moufang and Bol loops of small exponent

Gábor P. Nagy*

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Abstract. In this paper we consider the Burnside problems for the class of Moufang and Bol loops. We show that a free finitely generated Moufang loop of exponent 3 is finite and that the orders of the finite 2-generated Bol loops of exponent 2 are not bounded.

The (strong) Burnside problem asks for the finiteness of the free k-generated group $B(k,p)$ of prime exponent p. The answer is known to be negative in general. However, the answer to the restricted Burnside problem is affirmative: for each k and p, there exist a positive integer $b_{k,p}$ such that any *finite* k-generated group of exponent p has size at most $b_{k,p}$.

A. N. Grishkov [6] extended this last result for the class of *Moufang loops*, assuming $p \neq 3$. By a result due to R. H. Bruck [1], we know that commutative Moufang loops of exponent 3 are locally finite. In the first part of this paper, we solve the general Burnside problem with parameters $(k,3)$ for the class of Moufang loops showing that Moufang loops of exponent 3 are locally finite.

Bol loops are natural generalizations of Moufang loops. In the second part of this paper, using an example of E. Kolb and A. Kreuzer [10], we show that the restricted Burnside problem with parameters $k=2$, $p=2$ fails for this class. At the end of this section, we prove that a Bol 2-loop is solvable if and only if all of

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its 2-generated subloops are solvable (cf. the analogous result [13, Corollary 2] by J. G. Thompson for groups). Finally, we present three open problems motivated by our approach concerning involutorial Bol loops.

1. Moufang loops of exponent 3

The aim of this section is to show that Moufang loops of exponent 3 are locally finite. We also give a construction method for the free exponent 3 Moufang loops with k generators, using the free commutative Moufang loop of exponent 3 with $k+1$ generators.

In this paper we will try to use the most common terminology and notation of the non-associative generalization of group theory. For a loop $(L,\cdot)$, we denote by λ_x and ρ_x the left and right translations $\lambda_x(y)=xy$ and $\rho_x(y)=yx$, respectively. We define the *left translation group*

$$G(L)=\langle \lambda_x | x\in L\rangle$$

and the *multiplication group*

$$\mathcal{M}(L)=\langle \lambda_x,\rho_x | x\in L\rangle$$

of L.

Among non-associative structures, the most important ones are probably the *Moufang loops*. These can be defined by several identities; two equivalent ones are

$$x\cdot(y\cdot xz)=(xy\cdot x)\cdot z,\qquad (xy\cdot z)\cdot y=x\cdot(y\cdot zy).$$

A slight modification of the first identity,

$$x\cdot(y\cdot xz)=(x\cdot yx)\cdot z,$$

defines the larger class of (*left*) *Bol loops*. An important subclass of this is the class of *Bruck loops*, which is defined by the automorph inverse property

$$(xy)^{-1}=x^{-1}y^{-1}$$

and by the requirement that the map $x\mapsto x^2$ be a bijection of the underlying set. We use the notation $x\mapsto x^{1/2}$ for the inverse of $x\mapsto x^2$.

Bruck loops are called B-loops by G. Glauberman (see [5]); we follow his terminology. Since Moufang loops have the inverse property $(xy)^{-1}=y^{-1}x^{-1}$, the automorph inverse property implies commutativity in their class. The class of commutative Moufang loops can be characterized by the identity

$$(xy)(xz)=x^2(yz).$$

An interesting subclass of this class is the one of the commutative Moufang loops of exponent 3; we abbreviate this class by CML3.

The class of B-loops is strongly related to groups of odd order.

Definition. We call the automorphism σ of the group G a *Glauberman automorphism*, if the following conditions are satisfied.

(i) $\sigma^2 = 1$.

(ii) The set $S = \{x \in G | \sigma(x) = x^{-1}\}$ of anti-fixed elements of σ generate G.

(iii) The anti-fixed elements have finite odd order.

The following construction is a reformulation of the one from [5]. Let G be a group with Glauberman automorphism σ. We define the map

$$\mathrm{pr}\colon G \to S, \qquad x \mapsto (x\sigma(x^{-1}))^{1/2}.$$

Then $(S, \circ)$ with

$$x \circ y = \mathrm{pr}(xy) = (xy^2x)^{1/2}, \qquad x, y \in S$$

is a B-loop. Conversely, let $(L, \circ)$ be a B-loop. The automorph inverse property implies $J\lambda_x J = \lambda_x^{-1}$, where J is the inversion map $J\colon x \mapsto x^{-1}$. Hence, J induces an involutive automorphism of the left translation group $G(L)$ which turns out to be a Glauberman automorphism. Moreover, the B-loop arising from the pair $(G(L), J)$ is precisely the original $(L, \circ)$.

CML3 loops are obviously B-loops at the same time. Moreover, for an element $x \in L$, $x^3 = 1$ means precisely $\lambda_x^3 = 1$. Conversely, let G be a group with Glauberman automorphism σ and let us assume that $x^3 = 1$ for all $x \in S$. Taking arbitrary $x, y \in S$, we obtain $x^{-1}y = xy'x^{-1}$ with $y' = xyx \in S$. This means $(x^{-1}y)^3 = 1$ and

$$x \circ y = x^{-1}yx^{-1} = y^{-1}xy^{-1} = y \circ x.$$

Hence, we obtain a commutative B-loop, thus a commutative Moufang loop of exponent 3.

Lemma 1. *Let G be a group with Glauberman automorphism σ and let S be the set of anti-fixed elements. Let us define the B-loop $(S, \circ)$ as above. Then the elements $x_1, \ldots, x_k \in S$ generate G as a group if and only if they generate $(S, \circ)$ as a B-loop.*

Proof. The 'if' part is easy, since if $(S, \circ)$ is generated by $x_1, \ldots, x_k$ then S lies in the subgroup generated by $x_1, \ldots, x_k$, and $G = \langle S \rangle$.

For the 'only if' part, let us suppose $G = \langle x_1, \ldots, x_k \rangle$ and take an element $y = x_{i_1} \cdots x_{i_n} \in S$. We have

$$y = \mathrm{pr}(y) = (x_{i_1} \cdots x_{i_n}^2 \cdots x_{i_1})^{1/2} = x_{i_1} \circ (x_{i_2} \cdots \circ x_{i_n}),$$

hence $x_1, \ldots, x_k$ generate $(S, \circ)$ as a B-loop.

■

In [7], we showed how to relate groups with triality to *general* Moufang loops. The collineation group generated by the *Bol reflections* (see [4], [7]) turned out to be a group with triality. Conversely, we used three suitable classes of involutions of a group with triality in order to define the incidence structure of a Moufang 3-net.

The Bol reflection with axis $\xi = x$, $\eta = x$ and $\xi\eta = x$ will be denoted by $\sigma_x^{(1)}$, $\sigma_x^{(2)}$, $\sigma_x^{(3)}$, respectively. Moreover, we put $\rho = \sigma_1^{(1)}\sigma_1^{(2)}$ and

$$\begin{aligned} S &= \langle \sigma_1^{(1)}, \sigma_1^{(2)}, \sigma_1^{(3)} \rangle \cong S_3, \\ M_0 &= \langle \sigma_x^{(i)} | i = 1, 2, 3,\ x \in L \rangle, \\ M &= \text{the direction preserving subgroup of } M_0. \end{aligned}$$

We have a few lemmas.

Lemma 2. *Let L be a Moufang loop with elements $x_1, \ldots, x_k \in L$. Then the following statements are equivalent*:

(i) $L = \langle x_1, \ldots, x_k \rangle$,

(ii) $\mathcal{M}(L) = \langle \lambda_{x_1}, \ldots, \lambda_{x_k}, \rho_{x_1}, \ldots, \rho_{x_k} \rangle$,

(iii) $M\langle \rho \rangle = \langle \rho, \sigma_1^{(1)}\sigma_{x_1}^{(1)}, \ldots, \sigma_1^{(1)}\sigma_{x_k}^{(1)} \rangle$,

(iv) $MS = \langle \sigma_1^{(1)}, \sigma_1^{(2)}, \sigma_{x_1}^{(1)}, \ldots, \sigma_{x_k}^{(1)} \rangle$.

Proof. Let us suppose (i) and put

$$H = \langle \lambda_{x_1}, \ldots, \lambda_{x_k}, \rho_{x_1}, \ldots, \rho_{x_k} \rangle.$$

We define the set

$$K = \{x \in L | \lambda_x, \rho_x \in H\}.$$

By the inverse property, we have $K^{-1} \subseteq K$ and the Moufang identities imply

$$\lambda_{xy} = \rho_y^{-1}\lambda_x\lambda_y\rho_y, \qquad \rho_{xy} = \lambda_x^{-1}\rho_y\rho_x\lambda_x,$$

which means $KK \subseteq K$. Thus, $K \le L$ and by $x_1, \ldots, x_k \in K$ we have $K = L$ and $H = \mathcal{M}(L)$.

Considering the orbit of the unit element (or the origin), one gets immediately the implications (ii) $\Rightarrow$ (i) (or (iii), (iv) $\Rightarrow$ (i)).

Let us now compare the groups $H_1 = \langle \rho, \sigma_1^{(1)}\sigma_{x_1}^{(1)}, \ldots, \sigma_1^{(1)}\sigma_{x_k}^{(1)} \rangle$ and $H_2 = \langle \sigma_1^{(1)}, \sigma_1^{(2)}, \sigma_{x_1}^{(1)}, \ldots, \sigma_{x_k}^{(1)} \rangle$. The element $\sigma_1^{(1)}$ inverts the generators of H_1, thus it normalizes H_1 and we have $H_2 = H_1\langle\sigma_1^{(1)}\rangle$. Or, differently said, H_1 is the subgroup of H_2, consisting of the elements inducing an even permutation on the set of directions. Since this holds for the subgroup $M\langle\rho\rangle$ of MS as well, we have (iv) $\Rightarrow$ (iii).

We now calculate the coordinates of the following collineations.

$$
\begin{aligned}
&\sigma_m^{(1)}\colon (x,y) \mapsto (mx^{-1}m, m^{-1}\cdot xy),\\
&\sigma_m^{(2)}\colon (x,y) \mapsto (xy\cdot m^{-1}, my^{-1}m),\\
&\sigma_m^{(3)}\colon (x,y) \mapsto (my^{-1}, x^{-1}m),\\
&\sigma_1^{(1)}\sigma_m^{(1)} = (\rho_m^{-1}\lambda_m^{-1}, \lambda_m),\\
&(\sigma_1^{(1)}\sigma_m^{(1)})^{\rho} = \sigma_m^{(3)}\sigma_1^{(3)} = (\lambda_m, \rho_m).
\end{aligned}
\tag{1}
$$

Assuming (ii), H_1 turns out to act transitively on the set of vertical lines. Since it permutes cyclically the three line classes, it even acts transitively on the whole set of lines. Thus, H_2 contains every Bol reflection (=the conjugacy class of $\sigma_1^{(1)}$), we have (iv).

■

Lemma 3. *Let $(L,\cdot)$ be a k-generated Moufang loop of exponent 3. Then the loop $(L,\circ)$ with $x\circ y = x^{-1}yx^{-1}$ can be embedded in a $(k+1)$-generated commutative Moufang loop $(S,\circ)$ of exponent 3. In particular, L is finite and we have $3|L| = |S|$.*

Proof. Let us put $\sigma = \sigma_1^{(1)}$, $\rho = \sigma_1^{(1)}\sigma_1^{(2)}$ and consider the group with triality $(M,\langle\sigma,\rho\rangle)$, associated to the 3-net coordinatized by $(L,\cdot)$. Let Σ be the set of all Bol reflections. For all $\tau_1,\tau_2\in\Sigma$, we have $(\tau_1\tau_2)^3 = 1$. Then $G = \langle\sigma\Sigma\rangle = M\langle\rho\rangle$ is a group with Glauberman automorphism σ and the set of anti-fixed elements is $S = \sigma\Sigma$. By (1), $x^3 = 1$ holds for all $x\in S$, hence the B-loop $(S,\circ)$ determined by the pair (G,σ) is CML3.

By Lemma 2, the elements $\rho, \sigma_1^{(1)}\sigma_{x_1}^{(1)}, \dots, \sigma_1^{(1)}\sigma_{x_k}^{(1)}$ generate the group $G = M\langle\rho\rangle$. By Lemma 1, they must generate $(S,\circ)$ as a commutative Moufang loop as well. So, by [1, Theorem VIII.11.3.] of Bruck, $|S| < \infty$ and $3|L| = |S|$. Finally, the map $x\mapsto \sigma_1^{(1)}\sigma_x^{(1)}$ is an $L\to S$ injection which is a homomorphism of B-loops at the same time.

■

Lemma 4. *Let B be a $(k+1)$-generated CML3 loop. Then there exists a k-generated Moufang loop $(A,*)$ of exponent 3 with $|L| = \frac{1}{3}|B|$.*

Proof. We assume without loss of generality that $\{y_0, \dots, y_k\}$ is a minimal generating set for B. Then there is a maximal (normal) subloop A of B such that $y_0 \notin A$, $y_1, \dots, y_k \in A$ and $|B : A| = 3$. Since commutative Moufang loops are B-loops, the inverse map $J : x \mapsto x^{-1}$ is a Glauberman automorphism of the left translation group $G(B)$ of B. We define the subgroup

$$H = \{g \in G(B) | g(A) \subseteq A\} \leq G(B);$$

we have $H \triangleleft G(B)$, $|G(B) : H| = 3$ and $G(B) = H\langle \lambda_{y_0} \rangle$. Moreover, we put

$$\begin{aligned} S_3 &= \langle \rho = \lambda_{y_0}, \sigma = J \rangle \leq \mathrm{Aut}(H), \\ \mathcal{C}_1 &= \{\sigma^{\lambda_x} | x \in A\} = \sigma^H, \\ \mathcal{C}_2 &= \{\sigma^{\lambda_x} | x \in y_0 A\} = \sigma^{\lambda_{y_0} H}, \\ \mathcal{C}_3 &= \{\sigma^{\lambda_x} | x \in y_0^2 A\} = \sigma^{\lambda_{y_0}^2 H}. \end{aligned}$$

Since B has exponent 3, we have $(\tau_i \tau_j)^3 = 1$ for all $\tau_i \in \mathcal{C}_i$ and $\tau_j \in \mathcal{C}_j$ $(i = 1, 2, 3)$. Thus by [7, Lemma 2.2.], (H, S_3) is a group with triality, let $\mathcal{N}$ be the associated Moufang 3-net. By definition, the elements of one parallel class of lines of $\mathcal{N}$ are indexed by A, hence we can choose A to be the underlying set of the coordinate loop $(A, *)$ of $\mathcal{N}$.

Since for all $m \in B$ holds $\sigma\sigma^{\lambda_m} = \sigma\lambda_m\sigma\lambda_m^{-1} = \lambda_n^{-2} = \lambda_m$, we get

$$H\langle \rho \rangle = G(B) = \langle \lambda_{y_0}, \dots, \lambda_{y_k} \rangle = \langle \rho, \sigma\sigma^{\lambda_{y_1}}, \dots, \sigma\sigma^{\lambda_{y_k}} \rangle$$

using Lemma 1. Applying now Lemma 2, we obtain that $(A, *)$ is generated by the elements $y_1, \dots, y_k$.

■

Remark. For a given B, the resulting $(A, *)$ depends on the subloop A and the element y_0. However, if we take A' and y_0' such that for an automorphism φ of B the equalities $\varphi(A) = A'$ and $\varphi(y_0) = y_0'$ yield, then the obtained groups with triality are isomorphic: $H' = \varphi H \varphi^{-1}$ and $S_3' = \langle \lambda_{y_0'}, J \rangle = \varphi S_3 \varphi_{-1}$. In this case, the associated Moufang 3-nets are isomorphic as well. Of course, $(A, *)$ is determined up to isotopism, however, the property of having exponent 3 is universal for the class of Moufang (or, even Bol) loops (cf. [11]).

Theorem 5. *Let us denote the free, k-generated exponent 3 Moufang loop by L_k. We claim that L_k is finite and its size is*

$$\frac{\textit{size of the free } (k+1)\textit{-generated CML3 loop}}{3}.$$

Moreover, L_k is a G-loop, i.e., it is isomorphic to all of its loop isotopes.

Proof. Let us denote by F_k the free, k-generated CML3 loop. It is obvious that L_k and F_k are uniquely determined up to isomorphism. Now, we give a construction for L_k, using F_{k+1}.

Let us take a free generating set $\{y_0, \ldots, y_k\}$ of F_{k+1} and denote by A the normal subloop generated by $y_1, \ldots, y_k$. Applying the construction of Lemma 4 for $B = F_{k+1}$, we obtain a k-generated Moufang loop $(A, *)$ of exponent 3 with size as given in the theorem. However, Lemma 3 implies L_k to be finite with order at most $\frac{1}{3}|F_{k+1}|$. Thus, the loop $(A, *)$ has to be L_k.

To prove the last sentence of the theorem, it is enough to mention, that the three defining properties (being k-generated, having exponent 3 and being a Moufang loop) of L_k are universal, thus any loop isotope of L_k has them. Moreover, the orders equal as well, thus the isotopes are free, hence isomorphic to L_k. ∎

Remark. The uniqueness of the construction is clear as well, for choosing another free generating set $\{y'_0, \ldots, y'_k\}$ of F_{k+1}, there is an automorphism $\varphi \in \mathrm{Aut}(F_{k+1})$ with $\varphi(y_i) = y'_i$ $(i = 0, \ldots, k)$.

At the end of this section, we give an example of a proper Bol loop of exponent 3. This example motivates the general and the restricted Burnside problem with parameters $k = 2$, $p = 3$ for the class of Bol loops. By the author's best knowledge, this problem is still open.

Let K be a field of characteristic 3 and let us define the product $\mathbf{x} \circ \mathbf{y}$ on K^4 by

$$\begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 + x_1y_2 - x_2y_1 \\ x_4 + y_4 + (x_1y_2 - x_2y_1)(x_3 - y_3 + x_1y_2 - x_2y_1) \end{pmatrix}.$$

Then $(K^4, \circ)$ is a proper Bol loop of exponent 3.

2. Involutorial Bol loops

In this section, we show that the restricted Burnside problem with parameters $k = 2$, $p = 2$ fails for the class of Bol loops, that is, the orders of the 2-generated Bol loops of exponent 2 are not bounded. For this, we use the following example of E. Kolb and A. Kreuzer [10].

Let K be an arbitrary field with a field automorphism $x \mapsto \bar{x}$ of order 2. Let $R = K[[u]]$ be the ring of formal Laurent series over K. R is a local ring, any ideal of it has the form (u^n), let us put $R_{(n)} = R/(u^n)$. We denote the (coefficient-wise) extension of the K-automorphism to R and $R_{(n)}$ by $x \mapsto \bar{x}$ as well. Let us define the operation

$$x \oplus y = \frac{x+y}{1+u\bar{x}y}.$$

One immediately sees that $0 \in R$ is a unit element. The Bol property can be checked easily, see [10]. Moreover, if K has characteristic 2, then $x \oplus x = 0$ and R and $R_{(n)}$ turns out to be an involutorial Bol loop (cf. [9, Kapitel 7]).

Let us now put $K = \mathbb{F}_4$ with the usual Frobenius automorphism. We need two simple properties of the loops R and $R_{(n)}$. The first one is the observation, that for any ring element d with $d\bar{d} = 1$, the map $x \mapsto dx$ defines a loop automorphism.

The second property concerns the center of the loops. A short calculation gives $Z(R) = \emptyset$ and

$$Z(R_{(n)}) = \{\alpha u^{n-1}; \alpha \in \mathbb{F}_4\} \cong \mathbb{Z}_2 \times \mathbb{Z}_2.$$

This immediately implies

$$R_{(n)} = R_{(n+1)}/Z(R_{(n+1)}).$$

Combining these two facts, we get that the automorphism group of $R_{(n)}$ acts transitively on $Z(R_{(n)}) \setminus \{0\}$. The proof of the next theorem can be compared with [1, Theorem 2.3.].

Theorem 6. *For any $n \in \mathbb{N}$, the involutorial Bol loop $(R_{(n)}, \oplus)$ is generated by two elements.*

Proof. We use induction on n, for $n = 1$ the situation is being clear. Assuming $n > 1$, we have $R_{(n-1)} = \langle \bar{x}, \bar{y} \rangle$ by hypothesis, where $\bar{x} = xZ$, $\bar{y} = yZ$, $x, y \in R_{(n)}$ and $Z = Z(R_{(n)})$.

Let us also suppose that $\langle x, y \rangle \neq R_{(n)}$ and take a maximal subloop K with $x, y \in K < L$. It is enough to show that $Z \leq K$, since this implies $\langle \bar{x}, \bar{y} \rangle \leq K/Z \neq R_{(n-1)}$, a contradiction.

Now, assuming $Z \not\leq K$ and putting $z \in Z \setminus K$, by the maximality of K, we have $\langle K, z \rangle = K\langle z \rangle = L$, hence $|L : K| = 2$ and K is a normal subloop (see [11, Corollary 3.3.]).

The above investigation of $Z(R_{(n)})$ shows that $\alpha z \in K$ holds for some element $\alpha \in \mathbb{F}_4 \setminus \{0, 1\}$. We have

$$\alpha z \oplus \alpha^2 z = (\alpha + \alpha^2) z = z \notin K,$$

which implies $z, \alpha z, \alpha^2 z \notin N = K \cap \alpha K$, hence $Z \cap N = \{0\}$. It is clear that N is a normal subloop of index 4, thus $R_{(n)} = N \times Z$ and $N \cong R_{(n-1)}$. However, this also means $|Z(R_{(n)})| > 4$, which is not the case.

■

The next proposition emphasizes the importance of 2-generated involutorial Bol loops. This proposition can be seen as an analogue of the deep result [13, Corollary 2] by J.G. Thompson.

Proposition 7. *Let L be a finite Bol 2-loop. Then L is solvable if and only if any 2-generated subloop of L is solvable.*

Proof. The 'only if' part of the statement is clear. For the 'if' part, because of [11, Theorem 3.2], we only have to show that L is universal Bol 2-loop. Take a principal isotope $(L, *)$

$$x * y = x/b \cdot a\backslash y$$

of L and an arbitrary element $x \in L$. The left translation map λ_x^* of $(L, *)$ is $\lambda_x^* = \lambda_u \lambda_a^{-1}$, where $u = x/b$. By assumption, the 2-generated subloop $H = \langle u, a \rangle$ of $(L, \cdot)$ is solvable, hence a universal 2-loop (cf. [11, Theorem 3.2]).

Let us consider the principal isotope $x \circ y = x \cdot a\backslash y$ of $(H, \cdot)$. Then the left translation map by $u \in H$ is precisely the restriction of the map λ_x^* to the set H. However, the product of two left translations orders the elements of a Bol loop in cycles of equal length (cf. [2, Theorem 3]), the $(H, \circ)$-order of u equals the order of the permutation λ_x^*. Therefore, $(H, \circ)$ being a universal 2-loop, the $(L, *)$-order of x turns out to be a power of two. Since we have chosen $x \in L$ arbitrarily, we obtain the solvability of L.

■

We close this section with three open questions about involutorial Bol loops.

Problem 1. *Are all finite involutorial Bol loops solvable?*

Problem 2. *Is the stabilizer $G(L)_1$ Abelian for any finite 2-generated involutorial Bol loop L?*

Problem 3. *Is any finite 2-generated involutorial Bol loop homomorphic image of an $(R_{(k)}, \oplus)$?*

Concerning Problem 1, S. Heiss [8] showed that the solvability of the translation group $G(L)$ implies the solvability of L and that $G(L)$ cannot be a finite simple group. Moreover, it follows form a result of P. Cameron and G. Korchmáros [3] that $G(L)$ cannot act 2-transitively on L. Obviously, a non-solvable finite involutorial Bol loop of minimal order would yield a finite simple proper Bol loop. On the Prague conference "LOOPS '99", O. Chein called the problem of existence of finite simple proper Bol loops the "Main Problem" of the theory of loops.

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G. P. NAGY, Mathematisches Institut der Universität Erlangen-Nürnberg, Bismarkstr. $1\frac{1}{2}$, D-91054 Erlangen, Germany; *Current address*: SZTE Bolyai Institute, Aradi vértanúk tere 1, H-6720 Szeged, Hungary; *e-mail*: nagyg@math.u-szeged.hu