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Book Reviews

Communicated by P. Hajnal

JOHN L. CASTI, **Mathematical Mountaintops**, 186 pages, Oxford University Press, Oxford, 2001.

What might be the better beginning of a book like this than a citation from Aristotle's *Metaphysics*? "Those who assert that the mathematical sciences say nothing of the beautiful are in error. The chief forms of beauty are order, commensurability and precision". After this beginning, in the Prologue the question is discussed whose essence is faithfully reflected by its title: "The art of the problem". This brief account sheds some light on the background that presumably motivated the author in choosing "the five most famous problems of all time" which he undertook to present in the book. Of course, the guide is Hilbert's famous list of 23 problems from 1900.

One can find further details about this list in Chapter 1 dealing with the tenth problem. Then, in the sequel various aspects of this problem is discussed. It is concluded with a brief summary, just like the other chapters as well. Here one finds a formulation of the problem as follows: "Does there exists a single algorithm that will settle the solvability of every Diophantine equation in a finite number of steps?" And also, the concise *Answer*: No such algorithm exists.

The intuitive content of the four-colour problem (Chapter 2) provides the possibility of a spectacular presentation which the author does not hesitate to exploit. Indeed, the text is completed by 14 figures and even by 3 nice colour plates. In the summary, to the question "Do four colours suffice to colour any nondegenerate, planar map?" the *Answer* is: »Yes, provided one accepts computer analysis as a "proof"«. This conditional formulation refers to the fact, briefly discussed within the chapter and in the prologue as well, that computer-assisted proofs raise a number of new problems. These are philosophical and concern the nature of mathematical truth. Hence the status of such proofs is debated yet.

Philosophical questions also occur in Chapter 3 in which the continuum hypothesis is outlined. Cantor, Gödel and Cohen are intrinsic participants here, but the name of Imre Lakatos, René Thom and Roger Penrose also occurs in connection with certain philosophical aspects. Of course, the four classical schools of thought: formalism, logicism, Platonism and intuitionism are also mentioned, together with their well-known chief representatives, and the corresponding four answers to the problem are outlined as well. In summary, the formulation of the problem is as follows: "Does there exist a level of infinity greater than the natural numbers and less than the reals?". The *Answer*: "This question is independent of the axiomatic framework customarily used in mathematical argumentation."

The intuitive content of the Kepler conjecture (Chapter 4) is also easy to illustrate (ten figures are here). Though it is almost four hundred years old (Kepler, 1611), its proof given by Thomas Hales and his graduate student Samuel P. Ferguson is the most recent achievement among all the problems discussed in the book (1998). Among others, the role of László Fejes Tóth, founder of the Hungarian school of discrete geometry, is stressed in connection with this achievement. The conjecture is that the densest packing of unit spheres in $\mathbb{R}^3$ has density $\pi/\sqrt{18}$. This density is attained by the "cannonball packing", or, with a bit less popular term, by the face-centered cubic packing. Now, by Hales' result, the conjecture is true. Nevertheless, here also, just as with the four-colour problem, there is some doubt of the validity of Hales's solution, since his work is based heavily on the use of computer.

The proof of Fermat's Last Theorem (Chapter 5) is a rather recent achievement as well. Here the more than three century history of the theorem is briefly outlined, together with the exciting struggle of Andrew Wiles to find the proof without any gap. Both the past and the contemporary mathematicians who considerably contributed to this outstandingly famous story are mentioned and their role is described as well.

In the Epilogue ("The Magnificient Seven") problems that can be considered the most important problems of the new century are briefly outlined. These are seven problems, the solution of which will be awarded by the Clay Mathematics Institute, by a prize of one million dollars for each. Finally, the book is ended by a collection of Suggested Readings, and an Index.

In summing up, the reader will find in this book a wonderful journey to the mountaintops mentioned in the title. The panorama disclosed by the expert guidance of the author can be warmly recommended to everybody!

Gábor Gévay (Szeged)

VICTOR J. KATZ (Ed.), **Using History to Teach Mathematics: An International Perspective** (The MAA Notes Series **51**), XII+261 pages, The Mathematical Association of America, Washington, 2000.

From the Preface: "This book continues the long tradition of relating the history of mathematics to the teaching of mathematics. Throughout the twentieth century, mathematics educators at various levels have argued that the history of mathematics is a marvelous resource for motivating and exciting students studying mathematics." Accordingly, there has been a series of meetings devoted to this fruitful principle. The current volume is a collection of articles that are based on papers presented at the International Congress on Mathematical Education in Seville, Spain and at the Quadrennial Meeting of the International Study Group on the Relations Between History and Pedagogy of Mathematics, in Braga, Portugal, both in 1996. The result is a beautifully presented book whose contributors are international specialists in the history of mathematics and its use in teaching.

The book is divided into five parts with the following headings: General Ideas on the use of history in teaching; Historical ideas and their relationship to pedagogy; Teaching a particular subject using history; The use of history in teacher training; The history of mathematics. These headings indicate the wide scope of the book. Each part consists of several articles, from three to eleven. The articles, varying in technical or educational level, and also in the level of generality, provide a rich material of ideas, principles and approaches of the subject. The reader my browse to his taste in the various topics of mathematics, ranging from geometry to linear algebra, from combinatorics to the notion of pi. As for the appropriate historical view of the various aspects of mathematics, among others Constantinos Tzanakis calls the attention of the necessity to avoid both possible extremities. Namely, either the complete ignorance of history in the presentation of mathematics, or the naive attitude of following the historical development of a mathematical discipline as closely as possible, using original books, papers, and so on, is a method that has serious defects.

Of course, many great figures of the history of mathematics, or in a rather broad sense, of science, appear on the pages of the book. Only to mention some of them: Euclid and Liu Hui; Avicenna and Al-Biruni; Cavalieri and Beltrami; Descartes and Poncelet; Euler and Gauss; De Morgan and Hamilton; Hilbert and Poincaré; Minkowski and von Neumann. It is natural that at once on page 3 one finds the classical saying of Goethe: "*The history of science is science itself* ". It is a special pleasure to the reviewer that János Bolyai occurs in (at least) three articles (recall that we celebrate the bicentenary of his birth this year). On the other hand, it is disturbing that (on page 80) the data for his life are written as 1775–1856. Although his name is mentioned simply as "Bolyai", from the context it is clear that here *János* is meant, among other things as he is mentioned together with Lobachevsky in connection with the discovery of non-Euclidean geometry. But these data are of his father, *Farkas* Bolyai! Fortunately, the reader may find correct data for both the father and the son (1802–1860) e.g. in the article on the history of non-Euclidean geometry by Torkil Heide, pp. 201–211 and also, in the first article of the book by Siu Man-Keung, pp. 3-9.

The book is completed by numerous illustrations as well at the end by Notes on Contributors. To sum up, we may recommend it as an inspiring work especially for those teaching mathematics at any level, for students, and in general, for anybody who is interested in mathematics and its historical aspects.

Gábor Gévay (Szeged)

J. M. Aldous and R. J. Wilson, **Graphs and Applications, An introductory approach**, XII+444 pages and 1 CD-ROM (Windows), Springer-Verlag, London, 2000.

This is a nicely illustrated book containing the basics of graph theory in detailed description. The item was made for a regular course of the British Open University. The style and content of the book is determined by this fact. Students can easily use it for self-study. It is full of nice examples and exercises, and does not contain heavy proofs of deep theorems. The book is not only self-contained, but offers the solution of the provided exercises. Even more 'games' may be played by the attached *Graph Editor* and database of small graphs. This is assisted with the described *Computer Activities* in the book.

In the light of the above, this item is warmly recommended for undergraduate students willing to learn the basics of graph theory without perspiration.

János Barát (Szeged)

J.W. P. Hirschfeld (Ed.), **Surveys in Combinatorics, 2001. Papers from the 18th British Combinatorial Conference held in memory of Crispin Nash-Williams at the University of Sussex, Brighton, July 1–6, 2001** (London Mathematical Society Lecture Note Series **288**), X+301 pages, Cambridge University Press, Cambridge, 2001.

The prominent finite mathematics meeting *British Combinatorial Conference* was held for the 18th time in July, 2001. This book contains the survey articles written by the invited speakers. The frequent usage and citation of the past volumes shows that these papers are very good references. The quality of contributors and the discussed topics ensure that the present volume will continue this tradition:

J. Sheehan, *Crispin Nash-Williams*; M. Aigner, *The Penrose polynomial of graphs and matroids*; I. Anderson, *Some cyclic and* 1*-rotational designs*; A.R. Calberbank and A.F. Naguib, *Orthogonal designs and third generation wireless communication*; L.A. Goldberg, *Computation in permutation groups: counting and randomly sampling orbits*; B. Mohar, *Graph minors and graphs on surfaces*; M.S.O. Molloy, *Thresholds for colourability and satisfiability in random graphs and boolean formulae*; J.G. Oxley, *On the interplay between graphs and matroids*; J.A. Thas, *Ovoids, spreads and* m*-systems of finite classical polar spaces*; D.R. Woodall, *List colourings of graphs.*

János Barát (Szeged)

S. Jukna, **Extremal Combinatorics, With Applications in Computer Science**, XVII+375 pages, Springer-Verlag, Berlin – Heidelberg – New York, 2001.

Extremal Combinatorics is a part of finite mathematics devoted to questions of the following type: "How large/small can a set of objects be with some specific properties?" This kind of problems arise naturally in the theory of graphs/hypergraphs, number theory or in discrete geometry. Understanding the features of the extremal elements often reflects to the hole theory (e.g. via stability theorems).

The present book collects many different aspects of the field. It is wider than deep having 29 relatively short and independent chapters. These properties make the book accessible to a broad readership. Any graduate student in mathematics/computer science may enjoy a short trip to an interesting phenomena (e.g. by learning about the sunflower lemma) without much pre-knowledge. On the other hand researchers and teachers may look up the tools and techniques or handy proofs of basic theorems.

Beyond classical combinatorial techniques, the book includes the core of Extremal Combinatorics, among others: Turán's theorem, the Erdős-Ko-Rado theorem, chains and antichains, blocking sets in affine and projective planes, Paley graphs, designs, applications of the linear algebra method, Van der Waerden's theorem, Lovász local lemma, dominating and independent sets in graphs, Ramsey numbers, random walks, the Hales-Jewett theorem.

As the books on Combinatorics nowadays pet us, this volume also contains a large number of well-chosen exercises of various range of difficulty. There is a useful home page edited by the author which contains fragments of the book, list of errata, hints/solutions etc.:

lovelace.thi.informatik.uni-frankfurt.de/~jukna/EC_Book/index.html

We warmly recommend this well-written and nicely edited book to anybody with combinatorial interest.

János Barát (Szeged)

J. Matoušek and J. Nešetřil, **Invitation to Discrete Mathematics**, XVI+410 pages, The Clarendon Press, Oxford University Press, New York, 1998, Reprint 1999.

To make the best publicity for this item, we must quote a sentence from the preface describing a distinctive feature of the book: JOY.

"*... our text might help some readers to develop some positive feelings towards mathematics that might have remained latent so far.*"

Discrete Mathematics is today a regular part of the university curricula. This book concentrates especially on Combinatorics and Graph Theory. Any collection of subfields is of course subjective. The authors selected the areas with a long time teaching experience behind them. They also give details about what and how much one can include in a 13-week course with a 90-minute lecture and a 90-minute tutorial per week. We believed the authors and tried to lecture some areas of Combinatorics (combinatorial counting and generating functions) according to their text. We can report on a successful attempt. The students liked the detailed explanations and could follow the ideas well.

For the sake of completeness we list the chapters of the book: 1. Introduction and basic concepts; 2. Combinatorial counting; 3. Graphs: an introduction; 4. Trees; 5. Drawing graphs in the plane; 6. Double-counting; 7. The number of spanning trees; 8. Finite projective planes; 9. Probability and probabilistic proofs; 10. Generating functions; 11. Applications of linear algebra.

We can warmly recommend this item. Readers will enjoy the lively and informal style of the book. The beautiful illustrations and figures also increase the aesthetic experience. To maintain and improve the book, the first author collects the errors with corrections on his home page: kam.mff.cuni.cz/~matousek/IDM/index.html

János Barát (Szeged)

Attila Nagy, **Special Classes of Semigroups**, VIII+269 pages, Kluwer, Dordrecht – Boston – London, 2001.

The class of commutative semigroups is one of the best studied classes of semigroups. In some cases, the techniques developed for them can be extended to more general classes, sometimes entirely new methods are needed to generalize the results on commutative semigroups for wider classes. This monograph gives a systematic overview of the results known at present for certain classes of semigroups which generalize commutative semigroups. The classes considered in the book are the following: weakly commutative semigroups, $\mathcal{R}$-, $\mathcal{L}$-, $\mathcal{H}$-commutative semigroups, conditionally commutative semigroups, $\mathcal{RC}$-commutative semigroups, quasi commutative semigroups, medial semigroups, right commutative semigroups, externally commutative semigroups, E-m semigroups, exponential semigroups, WE-m semigroups, weakly exponential semigroups, (m, n)-commutative semigroups and $n_{(2)}$-permutable semigroups. The main topics studied in these classes are motivated by classical results on commutative semigroups: band decompositions, the structures of simple, archimedean, regular, inverse semigroups, the embeddability in groups and in unions of groups, special congruences and the subdirectly irreducible members.

This monograph is a good reference book for the experts working in this field of mathematics. It is also recommended to students interested in abstract algebra.

Mária B. Szendrei (Szeged)

Charles Robert Hadlock, **Field Theory and its Classical Problems**, XVI+323 pages, Cambridge University Press, Cambridge, 2000.

The presented book is a clear and concise introduction to classical results of Galois theory. Presupposing only basic knowledge in undergraduate level algebra and calculus it introduces the reader to topics such as the ancient problems of angle trisection, of duplication of the cube and of squaring the circle, the transcendence of e and of π, the construction of regular polygons, field extensions, the solution of polynomial equations by radicals, the fundamental theorems of Galois theory, Abel's Theorem, Hilbert's Irreducibility Theorem, the existence of polynomials over the rationals whose Galois group is the symmetric group of degree n. The author follows a historical pathway as he presents these topics. His historical remarks and comments make the reading more enjoyable. Exercises at the end of each section help the reader to acquire the material. Solution key to those exercises is also included. The book is an excellent reading for everyone, especially for instructors and first year graduate students in Galois theory.

László Zádori (Szeged)

Harald Niederreiter and Chaoping Xing, **Rational Points on Curves over Finite Fields Theory and Applications**, X+245 pages, Cambridge University Press, Cambridge, 2001.

For a long time, the study of algebraic curves over finite fields and their function fields was the province of pure mathematics. In the period 1977-1982, Goppa found applications of algebraic curves over finite fields with many rational points to coding theory. This, and the invention of elliptic-curve cryptosystems in 1985 created a strong interest in the area. Since 1995, the authors of this book opened new areas of applications; these include stream ciphers, hash functions, authentication schemes and construction of low-discrepancy sequences for quasi-Monte Carlo methods.

The main aim of this book is to make interested graduate students and researchers conversant with the recent developments and to provide the necessary background at the same time. The book favors the function-field viewpoint over the algebraic-geometry viewpoint of projective curves over finite fields.

Chapters 1 and 2 introduce the reader in the theory of algebraic function fields and in the class field theory of global function fields. Chapter 3 surveys explicit global function fields. In Chapter 4, the authors emphasize methods that lead to general results and are not just *ad hoc* techniques. Chapter 5 studies the asymptotic behavior of the number of rational places of global function fields when the genus tends to infinity. Chapters 6, 7 and 8 are devoted to applications. A compact discussion of the algebraic-geometric viewpoint and the theory of smooth projective curves over finite fields is presented in Appendix A.

The book is very clearly written. It is warmly recommended to anyone who is interested in nice mathematical theories and/or in the recent applications.

Gábor P. Nagy (Szeged)

Douglas F. Farenick, **Algebras of Linear Transformation** (Universitext), XIV+238 pages, Springer, New York – Berlin – Heidelberg – Barcelona – Hong Kong – London – Milan – Paris – Singapore – Tokyo, 2000.

From the text: "The aim of this book is twofold: (i) to give an exposition of the basic theory of finite-dimensional algebras at level that is appropriate for senior undergraduate and first-year graduate students, and (ii) to provide the mathematical foundation needed to prepare the reader for the advanced study of any one of several fields of mathematics."

"Therefore, this book begins where most linear algebra text have left off, and takes what is essentially a linear-algebraic approach to the study of algebras. The mathematical knowledge required of the reader is only elementary linear algebra and algebra."

The book consists of six chapters. The necessary background is in the first chapter. The aim of the second chapter is to introduce the basic notions that arise in the study of algebras. Among the topics studied in the first two chapters are duality, the Spectral Theorem, group algebras, quaternions, and the Frobenius-Pierce classification of finite-dimensional real division algebras. The theory of invariant subspaces is developed in the third chapter. The central result of this chapter, essential for students of advanced linear algebras, is Burnside's Theorem on invariant subspaces for proper subalgebras of the algebra of linear transformations. The theory of semisimple and simple algebras (or "Wedderburn theory") is developed in Chapter 4. Finite-dimensional operator algebras are an important class of semisimple algebras, and these are studied in Chapter 5. The final chapter of the book is an introduction to tensor products.

In each chapter several examples and 20-25 exercises are presented which help the reader in understanding the notions and results. This excellent book is highly recommended for senior undergraduate, first-year graduate students, teachers and researchers even if they are interested in different fields of mathematics.

László Szabó (Szeged)

Jean-Pierre Escofier, **Galois Theory**, XIV+280 pages, Springer-Verlag, New York – Berlin – Heidelberg – London – Paris – Tokyo – Hong Kong – Barcelona – Budapest, 2001.

This book is an excellent introduction to one of the most fundamental and most beautiful areas of abstract algebra: to Galois theory.

The book begins with a historical overview of the study of algebraic equations till the year 1600. Instead of just listing the milestones in the history of resolution of algebraic equations from the Babylonians to L. Ferrari, the author guides the reader through these centuries by helping him to imagine himself possessing only the notions, notation and techniques that mathematicians of these centuries had at their disposal. The theory of field extensions of finite degree and Galois theory is developed first for fields of complex numbers. The results presented here are applied to provide a criterion for solvability of algebraic equations by radicals and a criterion for constructibility of a point with ruler and compass. The assumption that the fields considered are subfields of the field of complex numbers is dropped only in the last chapters when finite fields and separable extensions are introduced. At the end of the book, two topics of current research is touched. The first is the inverse Galois problem which asks whether each finite group is the Galois group of an extension of the field of rational numbers. The second is a method for computing Galois groups which can be implemented on computers.

The book contains a lot of exercises and problems. Most of them are for practice and solutions — complete or sketchy — are provided. Others suggest interesting results beyond the scope of the text.

The author's main goal is formulated in the introduction as follows: 'The entire book was written with its student readers in mind, and with constant, careful consideration of the question of what these students will remember of it several years from now.' This goal is completely fulfilled. This is a well-written, self-contained textbook which is fascinating to read. I recommend it first of all to students and teachers but, in general, to everybody interested in this important subject of mathematics.

Mária B. Szendrei (Szeged)

Bruce E. Sagan, **The symmetric group. Representations, combinatorial algorithms, and symmetric functions.** Second edition (Graduate Texts in Mathematics **203**), XVI+238 pages, Springer-Verlag, New York, 2001.

This work is an introduction to the representation theory of the symmetric group. Unlike other books on the subject this text deals with the symmetric group from three different points of view: general representation theory, combinatorial algorithms and symmetric functions. The book consists of five chapters.

The first chapter is a short introduction to finite group representation theory with special emphasis on the methods of use when working with symmetric groups. (Maschke's theorem and Schur's lemma are stated on pages 16 and 22, respectively.) The definitions of irreducible representations and group characters are given, and Frobenius reciprocity, character orthogonality is also covered. Only a minimal knowledge of group theory is assumed.

The second chapter deals specifically with representations of the symmetric group. The author approaches the representation theory through Specht modules rather than by idempotents of the group algebra. An emphasis is placed on dominance order of partitions. The role of the Kostka numbers and standard and semistandard tableaux is developed. The construction of Young's natural representation is given.

The third chapter deals with important combinatorial algorithms which are related to the symmetric group. These include the jeu de taquin of Schützenberger and the Robinson-Schensted correspondence. The Frame-Robinson-Thrall hook formula is presented with a recent beautiful proof based on the Novelli-Pak-Stoyanovskii bijection. The chapter describes some of the earlier theorems of the book in a purely combinatorial manner.

Chapter four deals with symmetric functions. Schur functions are introduced, first combinatorially as the generating functions for semistandard tableaux and then in terms of symmetric group characters. The Littlewood-Richardson rule and the Murnaghan-Nakayama rule are proved in the end.

The final chapter contains applications of the material of the previous chapters. These include Stanley's theory of differential posets and Fomin's related concept of growths. Next come a couple of sections showing how groups acting on posets give rise to interesting representations that can be used to prove unimodality results. Finally, Stanley's symmetric function analogue of the chromatic polynomial of a graph is discussed.

This book is a digestable text for a graduate student and it is also useful for a researcher in the field of algebraic combinatorics for references.

Attila Maróti (Szeged)

V.V. ANDRIEVSKII and H.P. BLATT, **Discrepancy of Signed Measures and Polynomial Approximation**, XIII+438 pages, Springer Verlag, Berlin – Heidelberg – New York, 2002.

This book discuss in detail the discrepancy of signed measures, that is the $D[\sigma] := \sup\{|\sigma(J)| : J \text{ subarc of } L\}$ quantity, where L is a curve or arc and σ is a signed measure. Its various estimates are considered.

In the first chapter the short excerpt of the applied theories can be found: A summary of the (logarithmic) potentials which is a basic tool and the very language of this book is presented. Then it outlines the Riemann's conformal mapping theorem and the theories connected with and developed from it, like the Faber polynomials, the quasiconformal mappings and their important properties and the quasiconformal arcs and curves.

The second chapter begins with a discussion on the Jentzsch–Szegő theorem, which states that for every $\sum_{k=0}^{\infty} a_k z^k$ power series convergent on the unit disk, there exists such a sequence $\{n_k\}$ that for every sector $S(\alpha, \beta) = \{z : \alpha \le \arg z \le \beta\}$ the number of the zeros in $S(\alpha, \beta)$ of the n_kth partial sum, diveded by n_k, tends to $\frac{\beta-\alpha}{2\pi}$, as $k \to \infty$. Then the speed of this convergence is presented, that is, those Erdős–Turán type estimates. Then these estimates are reformulated in the terms of potentials, and are proved in case of quasiconformal arcs and curves. This reformulation connects the Jentzsch–Szegő and Erdős–Turán theorems with the newer results and creates the framework for more general theorems. The authors also examine the sharpness of these results and how these simplify if the arcs are smooth or real intervals.

In the next 3 chapters the authors show several estimates, mostly for "Lipschitz smooth" measures ($\sigma^+(J) \le M \cdot \mu_L(J)^\beta$ for all J subarc, where μ_L is the equilibrium measure of L) for quasiconformal, or smoother arcs. These results take into account the global behaviour (the upper and/or lower bounds of the supremum norm) of the U^σ potentials on the appropriate levels of the Green function of $\Omega = \mathbf{C}_\infty \setminus L$. Similar estimates are presented in the terms of the energy integral of σ.

In the further chapters the authors exhibit various applications. These applications mostly give the "rate" of the convergence of these distributions: the distribution of Fekete points in case of quasiconform and analytic curves, the distribution of zeros of orthogonal polynomials (on $[-1, +1]$ or $\mathbf{D} = \{z : |z| = 1\}$), the distribution sign changes of polynomial approximation in the $L_p(w(x)dx|_{[-1,+1]})$ norm.

The detailed proofs and the rich reference makes this book eligible for a self-study textbook and a reference book, too.

Béla Nagy (Szeged)

M. S. AGRANOVICH, YU V. EGOROV and M. A. SHUBIN (Eds.), **Partial Differential Equations IX** (Encyclopaedia of Mathematical Sciences **79**), IV+281 pages, Springer-Verlag, Berlin – Heidelberg – New York, 1997.

This EMS volume consists of three contributions: I. Elliptic Boundary Problems by

M. S. Agranovich, II. Boundary Value Problems for Elliptic Pseudodifferential Operators by A. V. Brenner and E. M. Shargorodsky, III. Elliptic Boundary Value Problems in Domains with Piecewise Smooth Boundary by B. A. Plamenevskij. The exhibited articles, written by leading experts, present nice surveys. They also contain the main ideas of the proofs, and some details where it is reasonable.

Part I. (pages 1-144) is devoted to the general linear elliptic boundary value problems on a smooth compact manifold M with boundary Γ. The classical BVP-s in a bounded domain G in $\mathbf{R}^n$ as special cases or examples are also considered. This part starts with the Scalar Elliptic Boundary Problems of kind:

$$Au = f \quad \text{on} \quad M_+ := M\backslash\Gamma, \quad B_j u = g_j \quad (j = \overline{1, q}) \quad \text{on} \quad \Gamma, \tag{1}$$

where A is a partial differential operator of order $m = 2q$ on M, and B_j are boundary partial differential operators of orders r_j. The coefficients of all differential operators are assumed to be C^∞ on M and, in general, complex-valued. Various notions of ellipticity of problem (1) are discussed, different forms of the Shapiro-Lopatinskij Condition and their consequences are explained. Then Sobolev spaces $H_s(\mathbf{R}^n_+)$ are introduced for arbitrary $s \geq 0$, the problems of restriction and extension from $H_s(\mathbf{R}^n)$ to $H_s(\mathbf{R}^n_+)$, and conversely, are studied, the interpolation inequality and the trace inequality (for $s > 1/2$) are presented and used for the theory of $H_s(M)$ and $H_s(\Gamma)$ spaces. Further a short proof of the main a priori estimate

$$\|u\|_{s,M} \leq C(s)\Big(\|Au\|_{s-m,M} + \sum_{j=1}^{q} \|B_j u\|_{s-r_j-\frac{1}{2},\Gamma} + \|u\|_{0,M}\Big) \tag{2}$$

is proved for the functions $u \in H_s(M)$-assuming, that problem (1) is elliptic, and

$$s \geq m = 2q \quad \text{and} \quad s > r_j + \frac{1}{2} \quad (j = \overline{1, q}); \tag{3}$$

where the term $\|u\|_{0,M}$ may be omitted if uniqueness holds for the problem (1). Later the author builds parametrices, studies the Fredholm Property of Elliptic Operators, and Smoothness of solutions. Elliptic Problems:

$$A(\lambda)u = f \quad \text{on} \quad M_+, \quad B_j(\lambda)u = g_j \quad j = \overline{1, q} \quad \text{on} \quad \Gamma \tag{4}$$

with a parameter $\lambda \in \Lambda$, where Λ is a fixed close angle (angular domain) on the complex plane with vertex at the origin also are considered. Here

$$A(\lambda) := \sum_{0 \leq l\tau \leq m} \lambda^l A_{m-l\tau}, \quad B_j(\lambda) := \sum_{0 \leq l\tau \leq r_j} \lambda^l B_{j,r_j-l\tau}$$

with $\tau \in \mathbf{N}$ and with differential operators A_s of the order s; $m = 2q$; and boundary operators $B_{j,s}$ of the order s (instead of r_j). The unique solvability of the problem is established for any $\lambda \in \Lambda$ with $|\lambda|$ great enough and an analogy of estimate (2) is proved. The discussion is continued with the study of Adjoint Elliptic Boundary Problems, Green's formula and the range of the operator corresponding to a Normal Elliptic Boundary Problem. Further the reduction of elliptic boundary problems to equations on the boundary is presented, using the Calderon projectors among others. In the next section elliptic boundary problems for elliptic systems are considered, variational boundary problems are studied, results for the elliptic BVP-s in complete scales of Banach spaces are presented too. Then the author considers the operator A_B corresponding to the problem $Au = f$, $B_j u = 0$, $j = \overline{1,q}$-assuming that after replacing A by $A(\lambda) := A - \lambda I$ we obtain a boundary problem elliptic with parameter in an angle Λ with bisectrix $\mathbf{R}_-$. The parametrix for $A_B - \lambda I$ is built, some functions of operator A_B are introduced and studied (powers of A_B, kernel and trace of $e^{-\tau A_B}$ $\tau > 0$, $\tau \to 0$ among others). The last section of Part I. deals with the spectral properties of operators corresponding to elliptic Boundary Problems e. g. with various estimates for the counting function for the eigenvalues.

The theme of Part II. (pages 145-215) is closely related to the previous Volumes of EMS (see e. g. the work "Index Theorems" by B. V. Fedosov in Volume 65). The present article is devoted to analytic aspects of the theory of BVP-s for Elliptic Pseudodifferential Operators, and there is almost no discussion of the topological point of view here. The main feature is the study of the Pseudodifferential Operators with transmission property. The importance of this property is explained by the following circumstances. "Let M be a smooth manifold with boundary $\Gamma = \partial M$ and interior Ω, and let M be embedded in a smooth manifold M_0 without boundary. Let E_0 and E_0' be smooth vector bundles over M_0, and write $E_0|_M = E$, $E_0'|_M = E'$. Finally, let $A : C_0^\infty(E_0) \to C^\infty(E_0')$ be a pseudodifferential operator and $u \in C_0^\infty(E) \equiv \pi_\Omega C_0^\infty(E_0)$, where π_Ω is the restriction operator from M_0 to Ω. (In the simplest case M is the closure of a domain Ω in the euclidean space $\mathbf{R}^n$, $M_0 = \mathbf{R}^n$, $E_0 = E_0' = \mathbf{R}^n \times \mathbf{C}$, $A\colon C_0^\infty(\mathbf{R}^n, \mathbf{C}) \to C^\infty(\mathbf{R}^n, \mathbf{C})$, $u \in C_0^\infty(M, \mathbf{C})$.) In order to define the action A on u we have to extend the function u to M_0. We take the zero extension $e_\Omega u$ defined by $e_\Omega u = u$ on Ω and $e_\Omega u = 0$ on $M_0 \backslash \Omega$. In general, the function $e_\Omega u$ is not in $C_0^\infty(E_0)$ since it may have a discontinuity along Γ. Thus the function $Ae_\Omega u$ may not be C^∞ smooth. If we require the correlation $A_\Omega u \equiv \pi_\Omega A e_\Omega u \in C^\infty(E')$ to hold, we have to narrow either the set of operators or the set of functions. In the first case we consider pseudodifferential operators with the transmission property (see §§2,3), in the second one we take the functions supported in M and vanishing on Γ (see §4)." The article is divided into four paragraphs. In §1 spaces $\mathcal{E}(\Omega)$, $\mathcal{D}(\Omega)$, $\mathcal{E}'(\Omega)$, $\mathcal{D}'(\Omega)$, $S(\mathbf{R}^n)$, $S'(\mathbf{R}^n)$, Besov spaces $\mathcal{B}^s_{p,q}(\mathbf{R}^n)$ and the direct and inverse Fourier transforms $F, F^{-1}\colon S(\mathbf{R}^n) \to S(\mathbf{R}^n)$ are introduced (using the notations $\hat{u}(\xi) = F(u)(\xi)$, $\tilde{f}(x) \equiv F^{-1}(f)(x)$). The pseudodifferential operators $A\colon S(\mathbf{R}^n) \to S'(\mathbf{R}^n)$ (with the symbol $a(x,\xi)$) are defined by the formula

$$Au(x) \equiv a(x,D)u(x) := (2\pi)^{-n} \int_{\mathbf{R}^n} e^{ix\xi} a(x,\xi)(Fu)(\xi)d\xi, \tag{5}$$

where $a \in C^\infty(\mathbf{R}^{n_1} \times \mathbf{R}^{n_2})$, and

$$|D_x^\beta D_\xi^\alpha a(x,\xi)| \le C_{\alpha,\beta} \langle \xi \rangle^{r - \rho|\alpha| + \delta|\beta|} \qquad (x,\xi) \in \mathbf{R}^{n_1} \times \mathbf{R}^{n_2}, \tag{6}$$

$r \in \mathbf{R}$; $\rho, \delta \in [0,1]$ are fixed; $\alpha \in \mathbf{Z}_+^{n_2}$, $\beta \in \mathbf{Z}_+^{n_1}$ are arbitrary (i. e. $a \in S^r_{\rho,\delta}$), $\langle\xi\rangle := \left[1+\sum_{i=1}^n \xi_i^2\right]^{1/2}$. In §2 the above mentioned restriction on the pseudodifferential operator A (i. e. on the symbol $\underline{a}$) is presented for a simplest one dimensional case, when $a = a(\xi) \in S^{-1}_{1,0}(\mathbf{R}\times\mathbf{R})$ and $\Omega = \mathbf{R}_+$. Then for $d \in \mathbf{Z}$ the H_d spaces of all C^∞ functions f on $\mathbf{R}$ with the asymptotic behaviour $f(t) \sim \sum_{j\le d} s_j t^j$ as $|t| \to \infty$ are introduced which satisfy for all k, $N \in \mathbf{Z}_+$ the inequalities

$$|D_t^k[f(t) - \sum_{d-N<j\le d} s_j t^j]| \le C_{k,N}|t|^{d-k-N} \qquad |t| \ge 1. \tag{7}$$

Further, the more important properties of the classes H_d and $H := \cup_{d\in\mathbf{Z}} H_d$ are studied, the "Boutet de Monvel Theory" of pseudodifferential operators with the transmission property is built (the necessary tools: trace and potential operators and singular Green operators, composition formulae are introduced and used). A typical result is Theorem 2.35 that guarantees the existence of the parametrix Q of elliptic Green operator P of order $d \in \mathbf{Z}$ and class $r \in \mathbf{Z}$ (moreover Q is of order $-d$ and class $r-d$). In §3 Parameter-Dependent Pseudodifferential Operators are studied for solving Parameter-Dependent BVP-s. The last paragraph deals mainly with BVP-s for elliptic pseudodifferential operators without the transmission property: the "Theory of Vishik and Eskin" (with applications) is presented.

Part III. (pages 217-273) is divided into three paragraphs: §1. Boundary Value Problems in a Cone, §2. Boundary Value Problems in Domains with Conical Points on the Boundary, §3. Boundary Value Problems in Domains with Edges. §1. starts with the Dirichlet problem (8) in the angle $K := \{x = (x_1, x_2) \in \mathbf{R}^2 | r > 0, \omega \in (0,\alpha)\}$, where (r,ω) are polar coordinates:

$$(\Delta u)(x) = f(x) \quad x \in K; \quad u = g \quad \text{on} \quad \partial K\backslash\{0\}. \tag{8}$$

This problem with the change of variables $(x_1, x_2) \to (t = \ln r, \omega)$ is transformed into the Dirichlet problem in the strip $\prod := \mathbf{R}\times(0,\alpha)$:

$$(\partial_t^2 + \partial_w^2)v(t,\omega) = F(t,w) \quad (t,\omega) \in \prod, v(t,\alpha_\pm) = G_\pm(t) \quad t \in \mathbf{R}, \alpha_+ = \alpha, \alpha_- = 0. \tag{9}$$

Then using the Fourier transform

$$\hat{w}(\lambda) := (2\pi)^{-1/2}\int_{\mathbf{R}} e^{-i\lambda t} w(t)dt \quad \lambda \in \mathbf{R} \tag{10}$$

we obtain a BVP for the family of ODE-s:

$$(\partial_\omega^2 - \lambda^2)\hat{v}(\lambda,\omega) = \hat{F}(\lambda,\omega) \quad \omega \in (0,\alpha); \quad \hat{v}(\lambda,\alpha_\pm) = G_\pm(\lambda) \quad \lambda \in \mathbf{R}. \tag{11}$$

The author proves that for every $F \in H^l(\Pi)$, $G_\pm \in H^{l+3/2}(\mathbf{R})$ there exists a unique solution $v \in H^{l+2}(\Pi)$ of (9) and the estimate

$$\|v; H^{l+2}(\Pi)\| \le c\Big(\|F; H^l(\Pi)\| + \sum_\pm \|G_\pm; H^{l+3/2}(\mathbf{R})\|\Big) \quad l \ge 0 \tag{12}$$

is valid for the Sobolev norms. The unique solvability of (9) and an analog of (12) — in the terms of the Sobolev norms with the weight $\exp(2\beta t)$ — is also proved (assuming that $\beta \neq j\pi\alpha^{-1}$ $(j = \pm1, \pm2, \ldots)$). These norms lead to the equivalent ones

$$\|u; V_\gamma^l(K)\| := \Big[\sum_{|\alpha|\le l}\int_K r^{2(\gamma-l+|\alpha|)}|\partial_x^\alpha u(x)|^2 dx\Big]^{1/2} \tag{13}$$

in the original coordinates; where $\gamma = \beta + l - 1$. Proposition 1.3 says that for any $\gamma \neq j\pi\alpha^{-1} + l - 1$, $j = \pm1, \pm2, \ldots$ and any $f \in V_\gamma^l(K)$, $g \in V_\gamma^{l+3/2}(\partial K)$ problem (8) has unique solution $u \in V_\gamma^{l+2}(K)$ and

$$\|u; V_\gamma^{l+2}(K)\| \le C(\|f; V_\gamma^l(K)\| + \|g; V_\gamma^{l+3/2}(\partial K)\|). \tag{14}$$

Next the author considers general elliptic problems in an open cone $K \subset \mathbf{R}^n$ with boundary ∂K and vertex 0. It is supposed that K cuts out on the unit sphere S^{n-1} (with center at 0) an open set Ω with smooth $(n-2)$-dimensional boundary $\partial\Omega$. Elliptic problems of the form:

$$\mathcal{L}(x, D_x)u(x) = f(x) \quad x \in K, \quad \mathcal{B}(x, D_x)u(x) = g(x) \quad x \in \partial K\backslash\{0\} \tag{15}$$

are considered, where $u = (u_1, \ldots, u_k)$, $f = (f_1, \ldots, f_k)$, $g = (g_1, \ldots, g_m)$ and $\mathcal{L}$, $\mathcal{B}$ are matrix differential operators with elements which are "model" operators. A scalar operator $\mathcal{P}(x, D_x)$ is called "model" differential operator if

$$\mathcal{P}(x, D_x) = r^{-l}\sum_{k=0}^{l} p_k(\omega, D_\omega)(rD_r)^k \equiv r^{-l}P(\omega, D_\omega, rD_r), \tag{16}$$

where $p_k(\omega, D_\omega)$ is a differential operator in Ω of order not higher than $l-k$ with coefficients smooth in $\overline{\Omega}$. Applying the Mellin transform problem (15) is transformed to the family of elliptic problems (for $v = (v_1, \ldots, v_k)$) with a parameter λ and the key a priori estimate (with $|\lambda|$) is given. Further, the structure of solutions $u \sim v$ is studied and the existence of fundamental solutions G to scalar elliptic BVP-s with normal boundary conditions is proven. Sharp estimates are given for the function G and its derivatives and the Oblique Derivative Problem in an Angle is discussed in details. Paragraphs 2,3 contain similar results (and techniques) as §1, with special attention to the behaviour of solutions to BVP-s near conical points on the Boundary (§2) or near edges on the Boundary (§3).

All three articles contain Bibliographical Notes and rich lists of References (Part I gives about 600 items).

Overall, it is a well written book, that contains comprehensive surveys with instructive examples, completed results and shows some directions for the generalization of the other results where it is possible. The book is warmly recommended to specialists in PDE-s, lecturers, physicists and also to students with the basic knowledge in PDE-s.

Jenő Hegedűs (Szeged)

N. G. Lloyd, W. M. Ni, L. A. Peletier and J. Serrin (Eds.), **Nonlinear Diffusion Equations and Their Equilibrium States 3** (Progress in Nonlinear Differential Equations and Their Applications **7**), X+572 pages, Birkhäuser, Boston – Basel – Berlin, 1992.

The book under review presents the Proceedings from a Conference held August 20-29, 1989 in Gregynog, Wales. It contains 37 refereed and revised scientific papers of the high level on Nonlinear Diffusion Equations and their Equilibrium States. The problems considered are interested both for the theory and applications. "Examples of current interest are biological and chemical pattern formation, semiconductor design, environmental problems such as solute transport in groundwater flow, phase transitions and combustion theory". The articles, published in this issue, are concentrated mainly around the equations $u_t = \Delta\varphi(u) + f(u)$ and $\Delta v + g(v) = 0$ respectively, a rich variety of the Problems is posed for these equations and the questions of the existence, uniqueness, regularity, blowup and development of singularities of the solutions are considered and solved. The absence of completely general methods for Nonlinear Equations and the character of the Applications lead to the importance of the study of special solutions such as positive or space-symmetric solutions. Many of the papers, in this book, present invaluable results connected with these special solutions, and they open new chapters in the qualitative theory of ordinary differential equations.

We present the complete list of the scientific papers published here: Liliane Alfonsi and Fred B. Weissler, Blow up in $\mathbf{R}^n$ for a Parabolic Equation with a Damping Nonlinear Gradient Term; Sigurd B. Angenent, Shrinking Doughnuts; F. V. Atkinson, Higher Approximations to Eigenvalues for a Nonlinear Elliptic Problem; Catherine Bandle, Positive Solutions of Emden Equations in Cone-Like Domains; Francisco Bernis, Nonlinear Parabolic Equations Arising in Semiconductor and Viscous Droplets Models; Michiel Bertsch and Roberta Dal Passo, A Parabolic Equation with a Mean-Curvature Type Operator; Fabrice Bethuel, Jean-Michel Coron, Jean-Michel Ghidaglia, and Alain Soyeur, Heat Flows and Relaxed Energies for Harmonic Maps; Gabrielle Bianchi and Henrik Egnell, Local Existence and Uniqueness of Positive Solutions of the Equation $\Delta u + (1 + \varepsilon\varphi(r))u^{(n+2)/(n-2)} = 0$, in $\mathbb{R}^n$ and a Related Equation; Marie-Françoise Bidaut-Veron, Singularities of Solutions of a Class of Quasilinear Equations in Divergence Form; Lucio Boccardo and François Murat, An Existence Result Via L^s-Regularity for Some Nonlinear Elliptic Equations; J. R. Cannon, Paul DuChateau, and Ken Steube, Identifying a Time-Dependent Unknown Coefficient in a Nonlinear Heat Equation; Kuo-Shung Cheng, On the Structure of Solutions for Some Semilinear Elliptic Equations; E. DiBenetto, J. Manfredi, and V. Vespri, A Note on Boundary Regularity for Certain Degenerate Parabolic Equations; Marek Fila, Josephus Hulsof, and Pavol Quittner, The Quenching Problem on the N-dimensional Ball; Bruno Franchi, Global Solutions for a Class of Monge-Ampère Equations; V. A. Galaktionov, M. A. Herrero, and J. J. L. Velázquez, The Structure of Solutions Near an Extinction Point in a Semilinear Heat Equation with Strong Absorption: A Formal Approach ; D. Hilhorst and H. J. Hilhorst, On a Conjecture by Hagan and Brenner; S. Kamin, L. A. Peletier, and J. L. Vázquez, A

Nonlinear Diffusion-Absorption Equation with Unbounded Initial Data; Hans G. Kaper and Man Kam Kwong, A Free Boundary Problem Arising in Plasma Physics; Bernhard Kawohl, Remarks on Quenching, Blow Up and Dead Cores; Tassilo Küpper and Charles A. Stuart, Bifurcation at Boundary Points of the Continuous Spectrum; Man Kam Kwong, A Comparison Result and Elliptic Equations Involving Subcritical Exponents; Howard A. Levine, Advances in Quenching; John L. Lewis and Andrew Vogel, On Some Almost Everywhere Symmetry Theorems; Yi Li, Symmetry Properties of Finite Total Mass Solutions of Matukuma Equation; J. B. McLeod, C. A. Stuart, and W. C. Troy, An Exact Reduction of Maxwell's Equations; Kevin McLeod, A General I-Theorem for Semilinear Elliptic Equations; F. Merle and L. A. Peletier, On Supercritical Phenomena; Wei-Ming Ni and Izumi Takagi, On the Existence and Shape of Solutions to a Semilinear Neumann Problem; Patrizia Pucci and James Serrin, Global Asymptotic Stability for Strongly Nonlinear Second Order Systems; Yuan-Wei Qi, The Existence and Asymptotic Behaviour of Similarity Solutions to a Quasilinear Parabolic Equation; Ana Rodríguez and Juan Luis Vázquez, Maximal Solutions of Singular Diffusion Equations with General Initial Data; Michael Struwe, The Evolution of Harmonic Maps: Existence, Partial Regularity, and Singularities; Takashi Suzuki, Two Dimensional Emden-Fowler Equation with Exponential Nonlinearity; Achilles Tertikas, Global Bifurcation of Positive Solutions in $\mathbb{R}^n$; Laurent Veron, Conformal Asymptotics of the Isothermal Gas Spheres Equation; Shoji Yotsutani, Chemical Interfacial Reaction Models with Radial Symmetry.

Summarising, it is a well-collected issue of papers written by well-known, and leading experts of the topics. The problems, considered, are still current, they guided the researchers in the last decade.

Research workers in PDE-s, ODE-s, and in applied mathematics, engineering, physics, chemistry, and biology will find much interest both for themselves and for their graduate students. The reviewer recommends this book to everyone who is interested in the theory and applications of differential equations. The book will be very useful also for any Mathematical Library.

Jenő Hegedűs (Szeged)

Constantine M. Dafermos, **Hyperbolic Conservation Laws in Continuum Physics** (Grundlehren der matematischen Wissenschaften **325**), XVI+443 pages, Springer, Berlin – Heidelberg – New York – Barcelona – Hong Kong – London – Milan – Paris – Singapore – Tokyo, 2000.

The study of gas dynamics gave birth to the theory of quasilinear hyperbolic systems in divergence form, commonly called hyperbolic conservation laws. This book gives a presentation of this theory from the standpoint of its "genetic" relation to Continuum Physics, in which material bodies are realized as continuous media, and so-called "extensive quantities", such as mass, momentum and energy, are monitored through the fields of their densities, which are related by balance laws and constitutive equations. This subject is in a state of active development, so it is not easy to write a book presenting "a panoramic view of the terrain", but the author of this book succeeded.

The chapter headings are: Balance Laws; Introduction to Continuum Physics; Hyperbolic Systems of Balance Laws; The Initial Value Problem: Admissibility of Solutions; Entropy and the Stability of Classical Solutions; The L^1 Theory of the Scalar Conservation Laws; Hyperbolic Systems of Balance Laws; Admissible Shocks; Admissible Wave Fans and the Riemann Problem; Generalized Characteristics; Genuinely Nonlinear Scalar Conservation Laws; Genuinely Nonlinear Systems of Two Conservation Laws; The Random Choice Method; The Front Tracking Method; Compensated Compactness.

By all means, this excellent monograph will be an indispensable tool for research workers in mathematical physics, hydrodynamics, and conservation laws.

László Hatvani (Szeged)

Jun Kigami, **Analysis on Fractals** (Cambridge Tracts in Mathematics **143**), VIII+226 pages, Cambridge University Press, Cambridge, 2001.

Analysis on fractals, a developing area of mathematics, has started with the work of physicists during the 1970s and 1980s. They tried to describe phenomena on fractals, such as heat diffusion and vibration of material with a fractal structure. However the problems could not be treated in the framework of classical analysis. In the middle of the 1980s mathematicians took up the subject and developed the mathematical tools needed for a rigorous treatment.

This book is an introduction to the subject written by one of the active researchers in the area.

After the Introduction which gives a fairly detailed account of the history of the area, Chapter 1 presents the standard facts about self-similar sets. Basic notions and results are presented in a clear, concise way.

The first constructions of Kusuoka and Goldstein have been probabilistic. Later the author of this book developed an alternative way which is called the analytical approach. The rest of the book is an elaboration of the latter method. The main idea is the following: suppose we want to define the Laplacian on the Sierpinski gasket. First we define Dirichlet forms and Laplacians on the finite sets that approximate the Sierpinski gasket. Then it is shown that by choosing a proper scaling these converge to an operator; this is called the Laplacian on the Sierpinski gasket. Using this approach the book studies Dirichlet forms, Laplacians, eigenvalues of Laplacians and heat kernels on a class of self-similar sets. Some new results are presented here for the first time.

Chapter 1 is recommended to everyone who wants to learn the basic facts about fractals, especially self-similar sets. The book itself is a self-contained introduction to a new, developing area of mathematics. It is recommended to those who would like to go from the basics to current research topics.

László I. Szabó (Szeged)

Ken Palmer, **Shadowing in Dynamical Systems. Theory and Applications** (Mathematics and Its Applications **501**), XIV+299 pages, Kluwer Academic Publishers, Dordrecht – Boston – London, 2000.

In this book the author develops the theory of hyperbolic sets both for diffeomorphisms and flows, with an emphasis on shadowing. An orbit $\{x_k\}_{k=-\infty}^{\infty}$ of a diffeomorphism $f\colon \mathbb{R}^n \to \mathbb{R}^n$ (that is, $x_{k+1} = f(x_k)$ for all k) is said to ε-shadow a δ pseudo orbit $\{y_k\}_{k=-\infty}^{\infty}$ of f (that is $\|y_{k+1} - f(y_k)\| \le \delta$ for all k) if $\|x_k - y_k\| \le \varepsilon$ for all k. The shadowing theorem gives sufficient conditions under which δ pseudo orbits of a diffeomorphism are shadowed by true orbits of the diffeomorphism or a nearby diffeomorphism. Shadowing can be used to prove that hyperbolic sets are robust under perturbation, that they have an asymptotic phase property and also that dynamics near a transversal homoclinic orbit is chaotic. It is also suitable to give computer-assisted proofs that computer orbits of certain "chaotic" dynamical systems arising in practice can be shadowed by true orbits for long times, that they possess periodic orbits of long periods and that they are really chaotic.

This book is warmly recommended not only for research workers but also for students of advanced graduate courses in dynamical systems familiar with calculus in Banach spaces and with the basic existence theory for ordinary differential equations.

Lászlό Hatvani (Szeged)

H. Bart, I. Gohberg and A. C. M. Ran (Eds.), **Operator Theory and Analysis** (Operator Theory: Advances and Applications **122**, The M. A. Kaashoek Anniversary Volume), XXXIX+433 pages, Birkhäuser Verlag, Basel – Boston – Berlin, 2001.

On the occasion of the 60th birthday of M. A. Kaashoek, one of the leading experts of operator theory and its applications, an international workshop has been organized at the Vrije Universiteit Amsterdam in November, 1997. The present volume contains the proceedings of this workshop. The scientific part consists of 16 research papers covering a wide range in analysis as, for example, operator matrices, matrix functions, spectral theory, interpolation theory and some problems concerning differential equations.

The scientific part is preceded by biographical materials. The book contains "Curriculum Vitae of M. A. Kaashoek", the list of his publications and personal reminiscences by three of his (former) PhD students.

Surely, a great number of readers, interested in applied mathematics and engineering will find material of his/her interest in this book.

Endre Durszt (Szeged)

Carlos S. Kubrusly, **Elements of Operator Theory**, XIII+527 pages, Birkhäuser Verlag, Boston – Basel – Berlin, 2001.

This volume is a selfcontained introduction to the basic concepts of functional analysis and operator theory. The approach is so elementary that it does not rely even on the theory of Lebesgue integral. The first three chapters provide a detailed discussion of the auxiliary concepts and results connected with sets, vector spaces and metric spaces. The fourth chapter is devoted to the study of Banach spaces, where linear and metric structures come to work together. Fully rigorous proofs are given for the three fundamental theorems: the Uniform Boundedness Principle, the Open Mapping Theorem, and the Hahn–Banach Theorem on the extension of bounded linear functionals. The geometric structure of Hilbert spaces is studied in the fifth chapter, where the concept of orthogonality comes to the scene. The spectra of operators, normal operators and their extensions are considered in the last chapter, which culminates in the Spectral Theorem of Compact Normal Operators, with an outlook to the non-compact case.

The author makes a successful effort to keep the reader motivated all through the text. The style of the presentation is clear and elegant. In each chapter the main body of the text is followed by a long list of problems, accompanied by hints and frequently by thorough discussions.

This work can be recommended to a wide range of readers, including senior undergraduate students and first-year graduate students in mathematics, physics, engineering, or other fields.

László Kérchy (Szeged)

Theodore W. Palmer, **Banach Algebras and the General Theory of *-Algebras, Volume II: *-Algebras** (Encyclopedia of Mathematics and Its Applications **79**), XII+823 pages, Cambridge University Press, Cambridge – New York – Oakleigh – Madrid – Cape Town, 2001.

This is the second volume of a two-volume monograph on the topic indicated in the title. The first volume has been reviewed in this *Acta* **59**/3-4 (1994) p. 722.

This second volume starts with Ch. 9 which is devoted to the theory of *-algebras without additional hypothesises. Various essentially algebraic hypothesises on *-algebras are studied in Ch. 10. In Ch. 11 mainly the results of Ch. 9 and Ch. 10 are translated to the case of Banach *-algebras. Ch. 12 deals with "Locally Compact Groups and their *-Algebras".

This volume (as well as the first one) is written in a lucid, easily readable style (for graduate students). A number of examples and historical comments enlights the abstract theoretical discussion. The volume contains a very rich "Bibliography" (on 109 pages!).

This two-volume monograph is warmly recommended to graduate students and researchers interested in Banach algebras and *-algebras.

Endre Durszt (Szeged)

E. Ramirez de Arellano, M. V. Shapiro, L. M. Tovar and N. L. Vasilevski (Eds.), **Complex Analysis and Related Topics** (Operator Theory: Advances and Applications **114**), VII+284 pages, Birkhäuser Verlag, Basel – Boston – Berlin, 2000.

This volume is a collection of 16 interesting recent works related to methods of complex analysis in operator theory and mathematical physics. We find mainly self-contained expository surveys on the topics of Q_p functions which give rise to interpolation between the classical Dirichlet space $\mathcal{D}_1$ and the maximal Möbius invariant function space called the Bloch space $\mathcal{B}$; on Clifford analysis in Poincaré space unifying widespread results on Atiyah–Singer type operators and Hodge–Dirac operators; on Lie superalgebras of supermatrices with related integrable systems and pseudodifferential operators; on completeness of holomorphic vector fields; on the connection between Bergman–Toeplitz and pseudodifferential operators; on Hankel operators in several complex variables; and on reproducing kernel Hilbert spaces, respectively. Also much attention is paid to Schrödinger equations, subelliptic operators, index problems, space–time duality, quaternion regular functions and Lie algebras in Fock spaces in the additional research articles.

The papers presented are elaborated versions of invited talks delivered at the International Symposium on Complex Analysis and Related Topics held in Cuernavaca (Mexico) to celebrate the 35th anniversary of the Center for Research and Advanced Study attended by 50 mathematicians.

The book is addressed to researchers and postgraduate students in operator theory with emphasis to differential operators, complex analysis both in finite and infinite dimensions, various branches of spaces of holomorhic functions and mathematical physics. It is also warmly recommended to theoretical physicists as well.

László L. Stachó (Szeged)

Vladimir Y. Rovenskii, **Foliations on Riemannian Manifolds and Submanifolds** (Progress in Mathematics **177**), X+284 pages, Birkhäuser, Boston – Basel – Berlin, 1998.

Foliations are partitions of topological spaces which are locally isomorphic in various aspects with the partition of a vector space into cosets of a subspace. They arise naturally in connection with vector fields without singularities, actions of Lie groups, submersions and fibrations and they have several more applications in various branches of mathematics. In the context of Riemannian manifolds, the important basic results are not far from the standard graduate material, however, they are mostly widespread in the literature. In the 1970s a considerable progress started on this area. The author, with his activity since 1985, can be regarded as one of the leading researchers in this development. His present monograph contains the results of nearly 700 papers written by about 300 geometers. However, it concentrates in a large part to to problems in his field of main interest: dimensional questions for foliations on Riemann manifolds.

The book is divided into two parts. Part 1 (Chapters I-IV) consists of surveys on fundamental results and concepts of foliations (not only in the setting of Riemannian Geometry), local and global rigidity and splitting properties of Riemann manifolds with foliations and introduces new variational methods and integral formulas. Part 2 (Chapters V-VII) is devoted to the study of submanifolds with generators as ruled, canal and tubular manifolds. The chief aims are to establish rigidity and Segre type decompositions of such manifolds. The work is completed by three appendices of independent interest, among which Appendix A written with the coauthor V. Topoganov treats great circle foliations and extremal theorems on Riemann manifolds with curvature bounded above and includes a generalization of Berger's minimal diameter theorem.

As for the style, Rovenskii combines quoted facts and outlined examples with fully proved theorems with good taste, which makes the readability of the huge material much easier.

This book can be recommended to all researchers and students both in mathematics and theoretical physics working with Riemann manifolds. Actually the basic graduate material is sufficient for an intelligent reading.

László L. Stachó (Szeged)

Morton D. Davis, **The Math of Money, Making Mathematical Sense of Your Personal Finances**, 199 pages, Copernicus Books, An Imprint of Springer-Verlag, New York – Berlin – Heidelberg – London – Paris – Tokyo – Hong Kong – Barcelona – Budapest, 2001.

This book is a very unusual and very entertaining introduction to financial mathematics to readers with a modest mathematical background.

Each chapter starts with a section entitled Test Your Intuition. A few multiple choice questions are presented; the reader is invited to guess the answer. A majority of the correct answers seem more or less paradoxical (depending on the mathematical sophistication of the reader). Then the chapter goes on to present the related basic notions and ideas and solves the problems posed at the start (occasionally simply gives the answer).

The list of the titles of the chapters gives a good indication of the range of topics touched upon in the book: Investment Strategies, Interest, Bonds, Mortgages, Retirement, The Psychology of Investing, A Mathematical Miscellany, Statistics, Options.

To give a flavour of the content we give a random sample of the problems or questions discussed.

In Chapter 1 a "sure" betting system is described that was published in *Esquire* magazine in 1940. This is a much funnier introduction to the notion of expected value than the usual dice and coin tossing problems.

What role does interest play in various applications such as Social Security, lottery, perpetuities and deferred taxes?

What are bonds? How does the value of a bond change? What are junk bonds?

What do I have to know about mortgages before taking out one? Here is the first question of the chapter: Suppose you take out a 40-year mortgage for $ 100,000 at 16 percent interest and repay it in equal, monthly payments. Estimate the total amount of money that you will repay. (The correct answer is about $ 600,000.)

How do you plan to spend the money you saved up after retirement? How long will it last?

Chapter 6 is on investor psychology. What is the proper price of a stock? Are there any patterns of market behaviour?

Chapter 7 gives a short introduction to game theory. Here is the second problem from this part: You have 10,000 diamonds of various sizes and are interested in selecting the largest. You examine the diamonds, one at a time in random order, and must decide to accept or reject a diamond immediately after you inspect it. If you ever reject the largest diamond or accept a diamond that isn't largest, you lose. How high can you make the probability of selecting the largest diamond? (The answer is: about 3/8.) Try to find out what a good strategy might be. The strategy given in the appendix is the following: initially you reject the first 36.8 percent of the diamonds and then choose the first diamond that is larger than all the diamonds that preceded it.

The last chapter is an introduction to options.

I warmly recommend the book to anyone who wants to learn the basic ideas of financial mathematics in an enjoyable way.

László I. Szabó (Szeged)

Michael Kohlmann and Shanjian Tang (Eds.), **Mathematical Finance, Workshop of the Mathematical Finance Research Project, Konstanz, Germany, October 5–7, 2000**, 374 pages, Birkhäuser Verlag, Basel – Boston – Berlin, 2001.

The year 2000 was the centenary of the publication of Bachelier's thesis which is considered the starting point of mathematical finance. On that occasion a workshop was held at the University of Konstanz with the aim of bringing together practitioners, economists and mathematicians to review recent developments.

This volume consists of 35 articles from the participants covering a wide range of topics. These include the following: portfolio selection strategies; optimal transaction strategies; financial market heterogeneity; optimal time of default; interest rate models; optimal investment-consumption strategies; control theory in financial decision making; installment options; capital asset pricing; mutual debts and graph theory; backward stochastic Riccati equations; hedging strategies; semilinear stochastic differential equations; financial market models; option pricing; stochastic control problems.

The volume well reflects the rich variety of mathematical tools that has been applied to tackle real world financial problems.

László I. Szabó (Szeged)

YU. V. PROKHOROV and V. STATULEVIČIUS (Eds.), **Limit Theorems of Probability Theory**, X+273 pages, Springer, Berlin, 2000.

The book is a very fine collection of five authoritative essays on five main directions in asymptotic probability, each giving a state-of-the-art overview as seen from Vilnius (and the neighboring cities of St. Petersburg and Bielefeld) in 1991, when the original Russian edition was published in Moscow. Of course, any insider appreciates the relevance, indeed, the importance of the Vilnius view. The nine-year delay with this English translation (a superb job, done by B. Seckler) is unessential: while all five fields continued to develop and grow in these nine years, the essayists make honest attempts to explore the very essence of their fields, and mostly with success. Thus, most later developments of note would easily fit into the framework described in each of the five directions.

Once it is asymptotic theory, it is only natural to begin with a research-level review of the central limit theorem (with uniform and nonuniform rates of convergence and asymptotic expansions), laws of large numbers and the law of the iterated logarithm for sequences of real-valued random variables, done in the first part *Classical-Type Limit Theorems for Sums of Independent Random Variables* (pp. 1–24, with 67 references) by V. V. Petrov.

The second part *The Accuracy of Gaussian Approximation in Banach Spaces* (pp. 25–111, with 214 references), written by V. Bentkus, F. Götze, V. Paulauskas and A. Račkauskas, is on the central limit theorem for sums $S_n = X_1 + \cdots + X_n$ of independent and identically distributed random elements $X_1, X_2, \ldots$ of a real separable Banach space. Four main methods of obtaining rates of convergence for various functions of S_n are reviewed, with various examples such as the central limit theorem in the Skorokhod space $D[0,1]$, and convergence rates in Prokhorov and bounded Lipschitz metrics are also discussed. Then the main ideas of obtaining corresponding asymptotic expansions are explained. Finally, a few interesting applications are treated, motivated by problems from nonparametric statistics. These concern empirical processes, Kolmogorov–Smirnov, Cramér–von Mises and L-statistics. The great strength of this part is the very fine sketches of the main arguments for obtaining proofs, not necessarily for the best available results, concentrating on the essence of the ideas involved.

Coming back to the real-valued case, exactly the same may be said about the third part *Approximation of Distributions of Sums of Weakly Dependent Random variables* (pp. 113–165, with 277 references) by J. Sunklodas, which discusses in detail three main methods for the estimation of the rate of convergence in the central limit theorem for sums $S_n = X_1 + \cdots + X_n$ of m-dependent, ψ-mixing, uniformly strong mixing, absolutely regular and strong mixing sequences $X_1, X_2, \ldots$ of random variables. This is done in the uniform, L_1, bounded Lipschitz and some other metrics. In fact, many new results of the author's are included with full proofs. The last chapter here is about extensions to the case of sums of random fields defined on a higher-dimensional integer lattice. The fourth part *Refinements of the Central Limit Theorem for Homogeneous Markov Chains* (pp. 167–183, with 27 references) by P. Gudynas may be viewed as an additional chapter to the third, for the case when the sequence $X_1, X_2, \ldots$ is a homogeneous Markov chain, satisfying some regularity conditions.

Finally, the fifth part *Limit Theorems on Large Deviations* (pp. 185–266, with 191 references) by L. Saulis and V. Statulevičius is devoted to results in large deviation theory when the basic approximation is normal, obtained by the cumulant method of the authors' for sums of independent but not necessarily identically distributed random variables and, in even greater detail, also for partial sums S_n of various mixing sequences $X_1, X_2, \ldots$ and Markov chains. The method is shown to extend to polynomials and U-statistics in independent, identically distributed random variables, and even to multiple Itô integrals and to certain spectral density estimates based on some stationary sequences. In a sense complementing Part IV, the last chapter exposes the cumulant method for obtaining central and other limit theorems with nonnormal limits and the associated rates of convergence for partial sums of Markov chains.

A name index, filling four full pages, and a subject index complete the book. People doing research in any of the areas above will find the volume extremely useful as a reference work that, in particular, makes hundreds of papers from Russian periodicals available for the first time. It will be practically indispensable for a graduate student beginning to do research in asymptotic probability.

Sándor Csörgő (Szeged)

PETER BÜRGISSER, **Completeness and Reduction in Algebraic Complexity Theory**, XII+168 pages, Springer-Verlag, New York – Berlin – Heidelberg – London – Paris – Tokyo – Hong Kong – Barcelona – Budapest, 2000.

Complexity usually defined in terms of Turing machines. However sometimes this universal model is too general. Computing the determinant or permanent of a given matrix is naturally done by algebraic manipulations on the entries. Several models of computation (and related notions like the class NP, completeness and so on) can be developed for algebraic computations. This was done by Valiant (around 1980) and Blum–Shub–Smale (about ten years later). In 1986 Valiant received the Nevanlinna prize for his work in complexity theory. His contribution to algebraic complexity played central role and was mentioned in the address on his work. Smale, the Field medal winner mathematician started to work on complexity theory in the 80's. He and his co-workers developed a nice theory of computation with simulating open problems. Blum, Cucker, Shub, Smale, *Complexity and real computation*, Springer-Verlag, New York-Berlin, 1998 summarizes their work.

Peter Bürgisser in his Habilitationsschrift in mathematics presents these exciting developments and adds some other interesting results including some of his own ones. The present book is the monograph version of his Habilitation Thesis.

The chapter headings are: Introduction, Valiant's Algebraic Model of NP-Completeness, Some Complete Families of Polynomials, Cook's versus Valiants's Hypothesis, The Structure of Valiant's Complexity Classes, Fast Evaluation of Representations of General Linear Groups, The Complexity of Immanants, Separation Results and Future

Directions, References, List of Notation, Index. Most of the titles are self-explanatory. We just mention that the main notion of chapter 7 is the immanant of a matrix. It is a natural common generalization of determinant and permanent, where we add the same terms, but the signs vary according to an irreducible character of S_n. Its complexity is an interesting question, and the author of the book has obtained the strongest result so far concerning it.

The present book is a nice presentation of a fascinating subject requiring deep algebraic tools, nice combinatorial ideas and a lot of insight.

Péter Hajnal (Szeged)

J.N. Mordeson and P.S. Nair, **Fuzzy graphs and fuzzy hypergraphs** (Studies in Fuzziness and Soft Computing), XIV+248 pages, Physica-Verlag, Heidelberg, 2000.

Fuzzy mathematics has a strong motivation from industrial engineering. The first author, the director of 'The Center for Research in Fuzzy Mathematics and Computer Science' is a leading researcher of the field. In this volume the concepts of fuzzy graphs and hypergraphs are discussed. The purpose was to collect an up to date account of the results. In this way one can look up relevant theories and methods if needed.

At the first sight a fuzzy graph is nothing else than a weighted graph. However this is not the case. The methods used in the book exhibit this fact well: besides graph theoretical methods much of analysis and algebra are used. In fact mostly the basic graph theoretical notions are required in the book. More sophisticated topics (like Ramsey theory, extremal graph theory) are not treated. The references are also one-sided, only some standard graph theory books are quoted, while the specialized articles in fuzzy theory are listed in huge mass. Fuzzy hypergraphs are discussed in the last chapter. This is a recently developing area of studies.

The book well represents the work that established this branch of fuzzy theory. We mention some names here addition to the two authors whose contribution is essential in the reviewed monograph: L.A. Zadeh, A. Rosenfeld, R.T. Yeh, S.Y. Bang, K.R. Bhutani, P. Bhattacharya, W.L. Craine, M. Delgado, M.S. Sunitha, A. Vijakayakumar, B. Ding, A. Kiss, E. Takeda.

The book can be well used as a source book for researchers in fuzzy theory. Hopefully it stimulates some researchers to build a wider bridge between fuzzy theory and graph theory.

Péter Hajnal (Szeged)

R. MacKenzie, M. B. Paranjape and W. J. Zakrzewski (Eds.), **Solitons: Properties, Dynamics, Interactions, Applications** (CRM Series in Mathematical Physics), XXI+312 pages, Springer-Verlag, Berlin – Heidelberg – New York, 2000.

The present CMR (Centre de recherches mathématiques, that was created in 1968 by the Université de Montréal) Volume consists of written versions of all talks presented at the workshop: "Solitons: Properties, Dynamics, Interactions, Applications" that was held at Queen's University, Kingston, Canada over the period July 20-26, 1997. The reader will find in this series articles on mathematical and numerical aspects of solitons, recent developments in string theory, and applications of solitons in nuclear and particle physics, cosmology and condensed-matter physics among others.

We present the complete list of articles of the Volume: Berry Phase and Dissipation of Topological Singularities (by Ping Ao and Xiao-Mei Zhu), Normal Mode Spectra of Multi-Skyrmions (by W. K. Baskerville, C. Barnes, and N. G. Turok), Standard-Model Dirac Particles Trapped in Flat (Noncompact) Higher Dimensions (by Ronald Bryan), Planar QED in Magnetic or Electric Solitonic Backgrounds (by Gerald Dunne), Collective Coordinates and Inequivalent Coset Space Quantizations (by Juan Pedro Garrahan and Martín Kruczenski), Spatial Solitons of the Nonlinear Schrödinger Equation of Arbitrary Nonlinearity with a Potential Hill (by Boris V. Gisin), Hairs on the Unicorn: Fine Structure of Monopoles and Other Solitons (by Alfred S. Goldhaber), A Model for Partially Polarized Quantum Hall States (by T. H. Hansson and U. Nilsson), Ordering Dynamics of Topological Defect Networks (by Mark Hindmarsh), Gauge Theory Description of Spin Chains and Ladders (by Yutaka Hosotani), Soliton Solutions of the Integrable Chiral Model in (2+1) Dimensions (by Theodora Ioannidou), String Winding Modes From Charge Nonconservation in Compact Chern-Simons Theory (by Ian I. Kogan), Holes in the Charge Density of Topological Solitons (by M. Kugler), From Two-dimensional Black Holes to sine-Gordon Solitons (by J. Gegenberg and G. Kunstatter), Solitons and Excition Superfluidity (by I. Loutsenko and D. Roubtsov), Quantum Effects on Higgs Winding Configurations (by Arthur Lue), Solitons and Their Moduli Spaces (by N. S. Manton), Deformed Skyrmions (by L. Marleau), The Large $-N_c$ Renormalization Group (by Nicholas Dorey and Michael P. Mattis), Instantons in Nonirreducible Representations of the Lorentz Group (by D. G. C. McKeon), Fermion Vacuum Effects on Soliton Stability (by Stephen G. Naculich), Soliton Solutions of the σ-Model and Disoriented Chiral Condensates (by Prasanta K. Panigrahi and C. Nagaraja Kumar), Dynamics of Topological Magnetic Solitons (by N. Papanicolaou), Fun with Baby-Skyrmions (by T. Gisiger and M. B. Paranjape), Skyrmions and Domain Walls (by B. M. A. G. Piette and W. J. Zakrzewski), Fun with Electroweak Solitons (by Edward Farhi, Jeffrey Goldstone, Arthur Lue, and Krishna Rajagopal), Neutral and Charged Spin Excitations in the Quantum Hall Ferromagnet (by Rashmi Ray), Quantum Corrections to Monopoles (by G. Chalmers, M. Roček, and R. von Unge), Nonabelian Dyons (by B. J. Schroers), Electroweak Baryon Properties in Soliton Models (by Norberto N. Scoccola), Solitons, Duality, and Supersymmetric Gauge Theories (by Alfred D. Shapere), Solitonic Strings and Knots (by R. A. Battye and P. M. Sutcliffe), Toward a String Formulation of Vortex Dynamics (by Elsebeth Schröder and Ola Törnkvist), Domain Walls in a Chern-Simons Theory (by Manuel Torres), Microphysics of Gauge Vortices and Baryogenesis (by Mark Trodden), On a Dual Standard Model (by Tanmay Vachaspati), From Skyrmions to the Nucleon-Nucleon Potential (by Jochen Wambach and Thomas Waindzoch), Two-dimensional Solitons at Finite Temperature (by M. Kacir and I. Zahed), Nontopological Structures in the Baby-Skyrme Model (by B. M. A. G. Piette and W. J. Zakrzewski).

The book is warmly recommended to physicists and also to mathematicians interested in mathematical physics and applications of solitons.

Jenő Hegedűs (Szeged)